

SCATTERING OF ELECTRON FROM FIVE DIMENSIONAL WAVE EQUATION

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ABSTRACT. The solution of the radial part of the five dimensional wave equation for the case of scattering of an electron in a Coulomb field has been considered and it is found that half integral quantum numbers appear in the solution which lends support to the idea that the influence of the fifth coordinate is somewhat analogous to the existence of the spin.

INTRODUCTION

Banerjee (1958) has shown that if one assumes that the electron obeys a five dimensional wave equation, then the given values of the angular momentum for such an electron are half integral; the appearance of half integers strongly suggests that the five dimensional wave equation incorporates the spin of the electron in a way not quite apparent on theoretical grounds.

In the present paper, the author intends to indicate that the solution of the radial part of the five dimensional wave equation for the case of scattering of electron in a Coulomb field contains half integral quantum numbers which again lends support to the idea that the influence of the fifth coordinate is the same as the existence of the spin.

THEORY

We consider the case of stream of electrons moving past a Coulomb field given by $-\frac{Ze}{r}$ where Ze is the charge of the source of the field and r is the distance of the electron from the source-point.

We consider the relativistic wave equation in five dimensional space-time similar to the Klein-Gordon equation, i.e.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial s^2} + \left(\frac{E^2 - m^2 c^4}{\hbar^2 c^2} - \frac{2EZe^2}{\hbar^2 c^2 r} + \frac{Z^2 e^4}{c^2 \hbar^2 r^2} \right) \psi = 0 \quad \dots (1)$$

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If we change over to polar coordinates

$$\begin{aligned}x &= r \sin \theta \cos \phi \sin \chi \\y &= r \sin \theta \sin \phi \sin \chi \\z &= r \cos \theta \sin \chi \\s &= r \cos \chi\end{aligned}$$

then equation (1) reduces to

$$\begin{aligned}&\frac{\partial^2 \psi}{\partial r^2} + \frac{3}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \cdot \frac{1}{\sin^3 \chi \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) \\&+ \frac{1}{r^2} \cdot \frac{1}{\sin^3 \chi \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{1}{r^2} \cdot \frac{1}{\sin^2 \chi} \frac{\partial}{\partial \chi} \left(\sin^2 \chi \frac{\partial \psi}{\partial \chi} \right) \\&+ \left(\frac{E^2 - m^2 c^4}{\hbar^2 c^2} - \frac{2EZe^2}{\hbar^2 c^2 r} + \frac{Z^2 e^4}{\hbar^2 c^2 r^2} \right) \psi = 0 \quad \dots (2)\end{aligned}$$

The solution ψ of (2) can be written in the form $\psi = \psi_r \cdot \psi_\theta \cdot \psi_\phi \cdot \psi_\chi$ where ψ_r is the solution of

$$\frac{\partial^2 \psi_r}{\partial r^2} + \frac{3}{r} \frac{\partial \psi_r}{\partial r} + \left[\frac{E^2 - m^2 c^4}{\hbar^2 c^2} - \frac{2EZe^2}{\hbar^2 c^2 r} + \frac{Z^2 e^4}{\hbar^2 c^2 r^2} - \frac{l(l+2)}{r^2} \right] \psi_r = 0 \quad \dots (3)$$

where l is a positive integer.

$$\text{If we put } \rho = kr, \quad \frac{E^2 - m^2 c^4}{\hbar^2 c^2} = k^2, \quad \frac{EZe^2}{k\hbar^2 c^2} = \alpha$$

$$\text{and } \frac{Z^2 e^4}{c^2 \hbar^2} = \beta, \text{ then equation (3) reduces to}$$

$$\frac{\partial^2 \psi_r}{\partial \rho^2} + \frac{3}{\rho} \frac{\partial \psi_r}{\partial \rho} + \left[1 - \frac{2\alpha}{\rho} + \frac{\beta - l(l+2)}{\rho^2} \right] \psi_r = 0 \quad \dots (4)$$

If we put $\psi_r = \rho^l e^{i\phi} F$, equation (4) reduces to

$$\frac{d^2 F}{d\rho^2} + \frac{2}{\rho} \left\{ l + \frac{3}{2} + i\rho \right\} \frac{dF}{d\rho} + \frac{2}{\rho} \left\{ \left(l + \frac{3}{2} \right) i - \alpha + \frac{\beta}{2\rho} \right\} F = 0$$

i.e.

$$\rho \frac{d^2 F}{d\rho^2} + \frac{2}{\rho} \left\{ l + \frac{3}{2} + i\rho \right\} \frac{dF}{d\rho} + 2 \left\{ \left(l + \frac{3}{2} \right) i - \alpha + \frac{\beta}{2\rho} \right\} F = 0 \quad \dots (5)$$

Putting $\rho = \frac{1}{2} iz$ in (5) we get

$$z \frac{d^2 F}{dz^2} + (2l+3-z) \frac{dF}{dz} - \left(i\alpha + l + \frac{3}{2} - \frac{\beta}{z} \right) F = 0 \quad \dots (6)$$

Neglecting the term $\frac{\beta}{z}$, as β is very small, we get from (6)

$$z \frac{d^2 F}{dz^2} + (2l+3-z) \frac{dF}{dz} - \left(i\alpha + l + \frac{3}{2} \right) F = 0 \quad \dots (7)$$

Two independent solutions of (7) are as follows

$$W_{1,2}(i\alpha + l + 3/2, 2l+3, z) \quad \dots (8)$$

In the case of Klein-Gordon equation, the equation for the radial wave function

$$\frac{\partial^2 \psi_r}{\partial r^2} + \frac{2}{r} \frac{\partial \psi_r}{\partial r} + \left[\frac{E^2 - m^2 c^4}{\hbar^2 c^2} - \frac{2EZe^2}{\hbar^2 c^2 r} + \frac{Z^2 e^4}{\hbar^2 c^2 r^2} - \frac{l(l+1)}{r^2} \right] \psi_r = 0 \quad \dots (9)$$

Proceeding exactly in the same manner as in the case of five dimensional continuum we get in this case two independent solutions

$$W_{1,2}(i\alpha + l + 1, 2l+2, z) \quad \dots (10)$$

Comparing (8) and (10) we notice that equation (10) goes over to equation (8) if $l \rightarrow l + \frac{1}{2}$. The addition of half to l in equation (8) suggests that the spin of the electron has entered into the formalism in suitable way.

In the case of scattering in a Coulomb field the radial part of the Schrodinger's equation is of the form

$$\frac{\partial^2 \psi_r}{\partial r^2} + \frac{2}{r} \frac{\partial \psi_r}{\partial r} + \left[\frac{2m}{\hbar^2} \left(E - \frac{Ze^2}{r} \right) - \frac{l(l+1)}{r^2} \right] \psi_r = 0 \quad \dots (11)$$

l being a positive integer.

The two independent solutions are

$$W_{1,2}(i\alpha + l + 1, 2l+2, z)$$

$$\text{where } k^2 = \frac{2mE}{\hbar^2}, \alpha = \frac{Ze^2}{\hbar v}, \rho = kr \quad \dots (12)$$

and

$$z = -2i\rho$$

If we add an extra space dimension to the Schrodinger's equation the radial part becomes

$$\frac{\partial^2 \psi_r}{\partial r^2} + \frac{3}{r} \frac{\partial \psi_r}{\partial r} + \left[\frac{2m}{\hbar^2} \left(E - \frac{Ze^2}{r} \right) - \frac{l(l+2)}{r^2} \right] \psi_r = 0 \quad \dots (13)$$

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Proceeding as before we get two independent solutions

$$W_{1, \frac{3}{2}}(i\alpha + l + \frac{3}{2}, 2l + 3, z) \quad \dots \quad (14)$$

Here we notice the same difference between equations (12) and (14) as in case of (8) and (10).

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